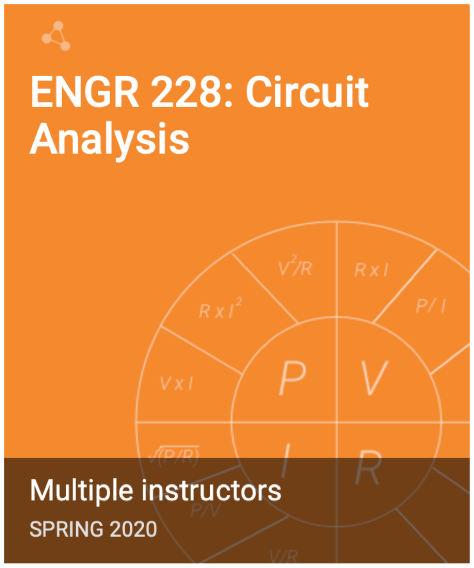


Chapter 5.5

Response of the RC circuit

Part II

Engr228 - Circuit Analysis

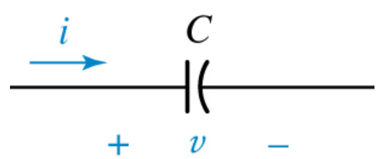
Spring 2020

Dr Curtis Nelson

Section 5.5 Part II Objective

- Learn to:
  - Analyze the transient and steady-state responses of RC circuits.

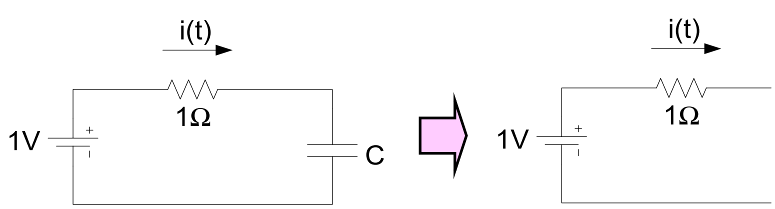
Capacitors



- The unit of capacitance is the Farad (F);
- 1 F = 1 Amp-Second/Volt = 1 Coulomb/Volt;
- The governing voltage and current relationship is:

$$\boxed{i_C(t) = C\frac{dv_C(t)}{dt}}$$

DC Characteristics of a Capacitor

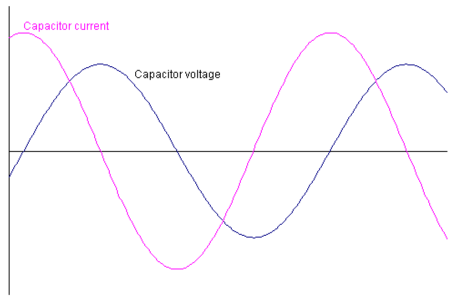


The capacitor acts like an "open circuit" at DC because the time rate of change of voltage is zero so, no current can flow through it.

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

Voltage-Current Relationship in a Capacitor

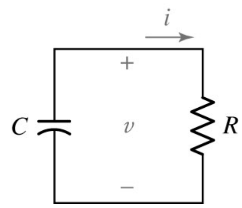
Current and voltage in a capacitor are not in phase with each other. **For sinusoidal waves, the voltage across a capacitor lags the current through it by 90º.** (In other words, the current leads the voltage by 90º.) In the diagram below, the tall purple waveform represents the current through a capacitor and the shorter blue waveform represents the voltage across a capacitor.



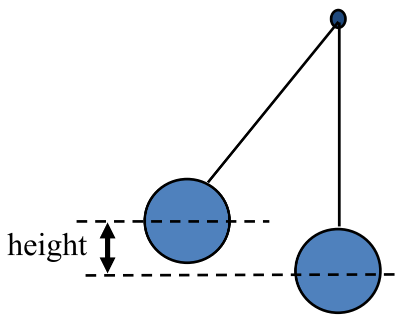
Types of First-Order Responses

- Circuits with one storage device (one capacitor or one inductor), are called **first-order circuits**.
- Their response to source excitations is composed of two parts:
  - Transient response, natural response, homogeneous solution (temporary position change)
    - Fades to zero over time.
  - Forced response, steady-state response, particular solution (permanent position change)
    - Follows the input;
    - Independent of time passed.

Source-Free RC Circuits



Capacitor C has energy stored so initial voltage is V_0 or V(0-)



Similar to a pendulum that is at a height h where the potential energy is nonzero.

Solving Source Free RC Circuits

Assume the solution is of the form $v(t) = Ae^{st}$
where A and s are the constants that need to be solved for.

Substitute $v(t) = Ae^{st}$ into the equation: $\frac{dv(t)}{dt} + \frac{v(t)}{RC} = 0$

$$v_C(t) = V_{C0} e^{-\frac{t}{RC}} = V_{C0} e^{-\frac{t}{\tau}}$$

Time Constant

The term *RC* is called the **time constant** and is denoted by the symbol τ (*tau*).

$$\tau_C = RC \qquad \text{Units: seconds}$$

One time constant is defined as the amount of time required for the output to go from its initial value V(0) to 36.8% of its initial value.

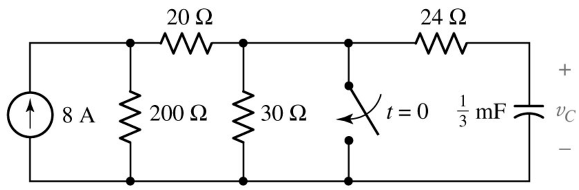
$$e^{-1} = 0.368$$

1^{st} Order Response Observations

- The voltage across a capacitor is the same *prior to* and *after* a switch at $t = 0$ seconds because this quantity cannot change instantaneously.
- Resistor voltage (or current) prior to the switch $v(0^-)$ can be different from the voltage (or current) after the switch $v(0^+)$.
- ***All*** voltages and ***all*** currents in an RC circuit follow the same natural response $e^{-t/\tau}$.

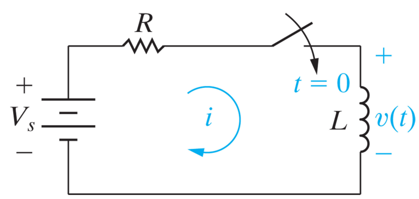
Example Problem

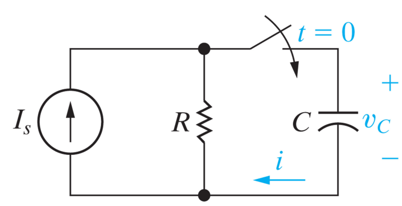
Find v_C(t) for all time in the circuit below.



$v_C(0) = 192V$ $\qquad v_C(t) = v_C(0)e^{-125t}$

Driven RC Circuits



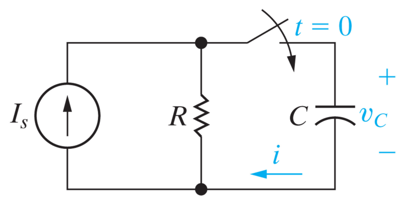


- Many RC circuits are driven by a DC or an AC source. The ***complete*** response of a driven RC circuit is the sum of the transient response and the forced response:

$$i(t) = tran(t) + forced(t)$$

Complete Solution for RC Circuits

The complete solution is found in section 5.5 of your text book:



$$v_c(t) = I_s R + (v_c(0) - I_s R)e^{-\frac{t}{RC}}$$

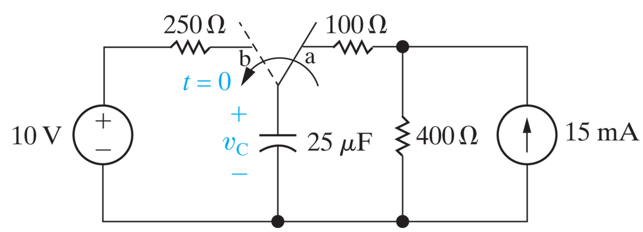
$$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-\frac{t}{RC}}$$

Procedure for Solving First-Order Circuits

1. Identify the variable of interest for the circuit. For RC circuits, it is best to choose the capacitive voltage.
2. Determine the initial value of the variable at $t = 0$. Note that if you choose capacitor voltage, it is not necessary to distinguish between $t = 0^-$ and $t = 0^+$ because this variable is time-wise continuous.
3. Calculate the final value of the variable as $t \rightarrow \infty$.
4. Calculate the time constant of the circuit.

Example Problem 7.52 (Nilsson 11th)

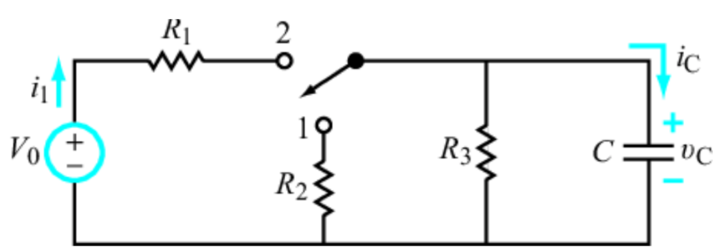
Find $v_c(t)$.



$v_C(t)=10 + (6 - 10)e^{-160t}$ V

Example 5.5.2 Zybooks (Natural Response)

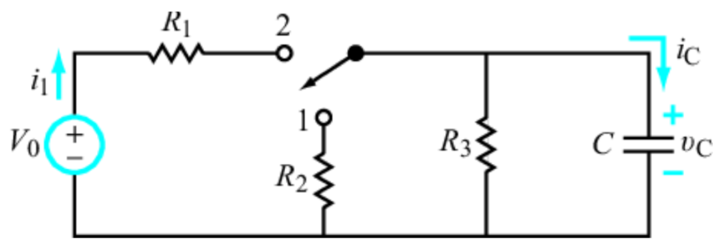
After having been in position 2 for a long time, the switch is moved to position 1 at t = 0. Given that V_0=12 V, R_1=30 kΩ, R_2=120 kΩ, R_3=60 kΩ, and C=100μF, determine v_C(t) for all time.



$v_c(t) = 8e^{-.25t}$

Example 5.5.1 Zybooks Total Response

After having been in position 1 for a long time, the switch is moved to position 2 at t = 0. Given that V_0=12 V, R_1=30 kΩ, R_2=120 kΩ, R_3=60 kΩ, and C=100μF, determine $v_C(t)$ for all time.



$$v_c(t) = 10(1 - e^{-.5t})$$

Section 5.5 Part II Summary

- You learned to:
  - Analyze the transient and steady-state responses of RC circuits.